

# An electric field in a gravitational field

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Received 19 August 2004, in final form 18 November 2004

Published 11 January 2005

Online at [stacks.iop.org/EJP/26/219](http://stacks.iop.org/EJP/26/219)

## Abstract

The behaviour of an electric field in a gravitational field is analysed. It is found that due to the mass (energy) of the electric field, it is subjected to gravity and it falls in the gravitational field. This fall curves the electric field, a stress force (a reaction force) is created, and the interaction of this reaction force with the static charge gives rise to the creation of radiation.

## 1. Introduction

In a recently published paper [1] Tartaglia and Ruggiero suggest that a unified approach, which emphasizes the similarities in handling electromagnetism and gravitation may be helpful in teaching both theories, and may help students to better assimilate concepts related to these theories.

In the present work we want to present another aspect of the connection between gravity and electrodynamics—the behaviour of an electric field (EF) in a gravitational field (GF). Actually, we consider the EF of a charge supported statically in a GF. This situation is connected to the equivalence principle (EP), according to which, a freely falling object in a uniform GF behaves like an object in free space, and a statically supported object in a GF behaves as if it is accelerated by an external force in a free space. According to classical electrodynamics, an accelerated charge radiates. Thus, using EP, one should expect that a supported charge in a GF will also radiate. Many physicists contradict this conclusion, arguing that the EP is not a general principle, and cannot be used to support the conclusion that a supported charge in GF radiates [2, 3].

In order to study this question without using EP, we first study the conditions at which electromagnetic radiation is created, and then investigate whether these conditions exist for a supported charge in a GF.

## 2. Radiation from an accelerated charge

In several papers published recently [4–6], the process of creation of electromagnetic radiation is analysed. For simplicity, we first investigated the most simple case of a linearly accelerated charge.

It is generally accepted that a reaction force created when a charge is accelerated is the force which is responsible for the creation of radiation. However, for many years it was believed that this reaction force is created by the radiation, and hence it was called ‘radiation reaction force’. However, at low velocities, the radiation is emitted in a plane perpendicular to the direction of motion and, hence, cannot impart any counter momentum to the accelerated charge, and thus no radiation reaction force can be created. This problem was known as the ‘energy balance paradox’ [7], and much effort was invested in finding what is the real reaction force which is responsible for the creation of the radiation.

In the above-mentioned papers [4–6] it is shown that the EF of an accelerated charge curves, and a stress force is created in this curved field. It is also found that this stress force is the main reaction force which is responsible for the creation of the radiation. The radiation reaction force acts in addition to the stress force, and it becomes significant only at very high velocities. The stress force, which is proportional to the acceleration of the charge, interacts with the accelerated charge, and an additional external force is needed to overcome this reaction (stress) force. The work performed by this additional external force, in addition to the work performed in creating the kinetic energy of the particle, is the source of the energy carried by the radiation. Calculation of the power created by this force yields exactly the Larmor formula for the power carried by the radiation (see appendix A for details). From these we find that the conditions needed for the creation of radiation are:

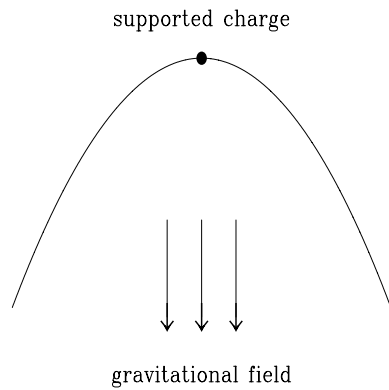
- (1) the existence of a relative acceleration between the charge and its EF;
- (2) curving of the electric field that creates the stress force which is the reaction force responsible for the creation of the radiation.

We also found that  $R_c$ , the radius of curvature of the curved EF, gives a measure for the characteristic wavelength of the radiation created [5]. Our next task now is to study whether conditions (1) and (2) do exist in the case of a charge supported at rest in GF.

It was also found that the stress force is inversely proportional to the radius of curvature,  $R_c$ , of the curved EF. Since  $R_c$  is inversely proportional to the acceleration, it comes out that the stress force is proportional to the acceleration of the charge. An additional bonus obtained by this approach is the justification found for the presence of the term  $\dot{a}$  ( $= da/dt$ ) in the equation of motion of the electric charge. This presence demands the supply of a third initial condition for the solution of the equation of motion, in addition to the two initial conditions (initial position and initial velocity) demanded in classical mechanics. The supply of the third initial condition—the initial acceleration, is justified as this acceleration supplies the information about the stress force, and this is needed to complete the description of all the forces involved in this configuration.

Analysing the process of the creation of radiation we found that the main feature which drives this process is the behaviour of the electric field of the accelerated charge. When the charge is accelerated, its electric field is not accelerated with the charge and, hence, lags behind the charge and becomes curved. This occurs because the EF, which is an independent physical entity, once induced by the charge on the space around it, is not attached to the charge anymore and its behaviour does not depend on the behaviour of the charge. When a charge is static, its electric field expands in space as concentric circles around the charge. But if the charge is moving, each circle is centred around the point it was induced from—actually, this is the way in which retarded potentials are calculated.

When the motion of the charge is of a constant velocity, the ratio between the velocity of the charge and the expansion velocity of the EF is constant. Hence, the field lines are straight lines, although they are not spread spherically symmetric around the charge (see [8]). If the



**Figure 1.** A curved field line of a charge supported in a homogenous gravitational field.

motion is of an acceleration, the distances between consecutive circles change continuously, and this is why the electric field lines become curved.

### 3. A charge in a gravitational field

After we defined the conditions needed for the creation of radiation by an accelerated charge, we turn to the case in which a charge is supported statically in a homogenous GF. In figure 1 we plot such a field line of a charge supported statically in a homogenous GF. The equations of the field lines for a homogenous GF were calculated by Rohrlich [9], and they are given in appendix B.

Usually, the immediate reaction of people to such a situation is this: ‘the situation is static, no work is performed in a static situation, and hence no radiation can be created’. However, the situation is not a static one, but rather a steady-state one, in which the EF of the static charge falls in the GF. We know that an EF of a charge is an independent physical entity, and once induced on space, its behaviour no longer depends on the behaviour of the charge that induced it (see [8]). The charge is supported and remains static. The EF of the charge is induced on space around the charge. The mass (energy) density of the EF is subjected to gravity, and it falls in the GF. Due to this fall, the EF becomes curved (as can be observed in figure 1), and a stress force is created in the EF. This stress force interacts with the static charge, and this is actually a reaction force that acts between the charge and its falling EF. This interaction causes a delay in the fall of the EF, braking it from being a free fall. In order to keep the EF in free fall, the GF has to perform extra work on the EF. This extra work is the source of the energy carried by the radiation. Since this extra work is performed by the gravitational source that creates the GF, this source loses energy—namely, the gravitational energy of the system becomes lower. The energy carried by the radiation is created at the expense of the gravitational energy of the system.

Thus, although the charge remains static, the system that contains the charge, its EF and the gravitational source is not static, and it does radiate. Extra work is performed by the gravitational source on the falling EF, which is converted to energy carried by the radiation.

It comes out that, indeed, the supported charge does not radiate (because it is static and no work can be performed on a static object). However, the configuration in which the charge is supported in a GF does radiate—the work that creates the radiation is performed by the source of the GF, and it is performed on the EF, against the reaction (stress) force, to maintain its free

fall. The energy loss, carried away by the radiation, is supplied by the object that performs the work—the gravitational field.

In this picture, the moving entity is the electric field, and the work is performed on this moving entity. If we now return to the EP, we find that a charge supported in a GF does not behave as a charge accelerated in a constant acceleration—the supported charge does not radiate. However, if we consider the basic question presented by the EP, ‘would a system of a supported charge in a GF behave as a system of a charge accelerated in free space’, the answer is positive: both systems radiate.

#### 4. Summary

The topic presented here is actually an extension of an earlier work [8]. In that work, the point was to show that fields are not only pedagogical or technical concepts that help to describe and calculate physical processes. Fields are independent physical entities, as Einstein has suggested [10], and they should be treated accordingly.

In the present work this concept is pushed further. When a charge is accelerated in free space, the stress force created in the curved EF of the charge interacts with the charge (which is the moving object), and overcoming this reaction force, radiation is created. When the charge is supported statically and its EF falls in the GF, the stress force interacts with the EF, and overcoming this reaction force, radiation is created.

Thus, if we relate the EP to the system charge + EF, we find that in both cases radiation is created and the EP is satisfied.

#### Acknowledgment

I would like to acknowledge suggestions and comments made by an anonymous referee which helped to improve the manuscript.

#### Appendix A

The angular distribution of the power,  $P$ , radiated by a linearly accelerated charge is given in the textbooks (see [11]):

$$\frac{dP}{d\Omega} = \frac{e^2 a^2}{4\pi c^3} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5} \quad (\text{A.1})$$

where  $\theta$  is the angle between the direction of motion and the direction of the emitted radiation. Integrating this formula over the angles yields:

$$P = \frac{2}{3} \frac{e^2 (\gamma^3 a)^2}{c^3} \quad (\text{A.2})$$

which for low velocities ( $\gamma \rightarrow 1$ ) yields

$$P = \frac{2}{3} \frac{e^2 a^2}{c^3}. \quad (\text{A.3})$$

This is the Larmor formula for the power carried by radiation from an accelerated charge.

#### Appendix B

The equations for the electric field of a charge supported in a homogenous gravitational field of strength,  $g$ , are given by Rohrlich [9] using cylindrical coordinates  $(\rho, z, \phi)$ :

$$E_\rho = \frac{8e\alpha^3 \rho u^2}{\xi^3} \quad (\text{B.1})$$

$$E_z = \frac{-4e\alpha^3 u u'}{\xi^3} (\alpha^2 (1 - u^2) + \rho^2) \quad (\text{B.2})$$

$$\xi^2 = (\alpha^2 (1 - u^2) - \rho^2)^2 + 4\alpha^2 \rho^2 \quad (\text{B.3})$$

where  $u^{-1} = \cosh(\sqrt{(1 - gz)^2 - 1})$  and  $u' = du/dz$ .  $\alpha = c^2/g$  is the radius of curvature of the electric field close to the location of the charge.

From these equations we can calculate the derivative of  $z$  with respect to  $\rho$  for the field lines of force (see Singal [12]). This expression cannot be integrated analytically. However, using this derivative, the line of force can be calculated numerically and drawn—this is how figure 1 was drawn.

## References

- [1] Tartaglia A and Ruggiero M L 2004 *Eur. J. Phys.* **25** 203
- [2] Will C M 1993 *Theory and Experiment in Gravitational Physics* (Cambridge: Cambridge University Press)
- [3] Ciufolini I and Wheeler J A 1995 *Gravitation and Inertia* (Princeton, NJ: Princeton University Press)
- [4] Harpaz A and Soker N 1998 *Gen. Rel. Grav.* **30** 1217
- [5] Harpaz A and Soker N 2001 *Found. Phys.* **31** 935
- [6] Harpaz A and Soker N 2003 *Found. Phys.* **33** 1207
- [7] Leibovitz C and Peres A 1963 *Ann. Phys.* **25** 400
- [8] Harpaz A 2002 *Eur. J. Phys.* **23** 263
- [9] Rohrlich F 1963 *Ann. Phys.* **22** 169
- [10] Einstein A and Infeld L 1938 *The Evolution of Physics* (New York: Simon and Schuster)
- [11] Jackson J D 1975 *Classical Electrodynamics* (New York: Wiley)
- [12] Singal A K 1997 *Gen. Rel. Grav.* **29** 1371